

**SIDHO-KANHO-BIRSHA UNIVERSITY, PURULIA**

**DEPARTMENT OF MATHEMATICS**



**Syllabus for Post Graduate Course in Mathematics**

**(Choice Based Credit System)**

**Effective from the Academic Session 2016-2017**

## **Syllabus of Post Graduate Course in Mathematics**

### **(Choice Based Credit System)**

P.G. course of studies in Mathematics shall be of two years duration divided into four Semesters: Semester I, Semester II, Semester III and Semester IV each of six months duration. Semester I, Semester II, Semester III and Semester IV examinations in Mathematics will be conducted at the end of each semester. Syllabus for P.G. course in Mathematics is hereby framed based on the following schemes and structures.

Using the guidelines of the University, Choice Based Credit System is offered. Core Courses should compulsorily be studied by all the students of the department. The major elective courses will run in Semester III and in Semester IV. For major elective courses a student would choose from a pool of courses (offered by the department) from his/her main subject of study. In case of open elective course, a student would choose from a pool of courses from other department(s). Outreach Programme is one kind of extension activity towards the society which will be helpful for a student in skill development. For add-on Course students have to choose a course from a pool of courses.

Total marks are 1200 of which each semester is of 300 marks. 20% marks are allotted for Internal Assessment for each theoretical paper. There is only one practical paper which is in Semester III. In Semester III, the first theoretical paper is general and has two special papers, the last but one theoretical paper is open elective paper and the last one is "Outreach Programme". In Semester IV, the first two theoretical papers are general and the next two papers are special papers, the fifth paper is the project work and the marks distribution for this is as follows: 25 Marks for written submission, 15 Marks for Seminar presentation and 10 Marks for Viva-Voce. Faculty members of the department will supervise the students for project work. The sixth paper is add-on course. In Semester III the department will offer a cluster of special papers and the students will have to choose special papers according to the norms to be decided by the Department. The corresponding papers are to be continued as special papers in Semester IV also.

**COURSE STRUCTURE**

<b>SEMESTER-I</b>	<b>Code</b>	<b>Course Title</b>	<b>Credit</b>	<b>Marks</b>	<b>No. of Class Hours /Week</b>
	MMATCCT101	Abstract Algebra	4	40+10	4
	MMATCCT102	Real Analysis	4	40+10	4
	MMATCCT103	Complex Analysis	4	40+10	4
	MMATCCT104	Ordinary & Partial Differential Equations	4	40+10	4
	MMATCCT105	Classical Mechanics & Calculus of Variations	4	40+10	4
	MMATCCT106	Topology	4	40+10	4

<b>SEMESTER-II</b>	<b>Code</b>	<b>Course Title</b>	<b>Credit</b>	<b>Marks</b>	<b>No. of Class Hours /Week</b>
	MMATCCT201	Linear Algebra	4	40+10	4
	MMATCCT202	Functional Analysis	4	40+10	4
	MMATCCT203	Numerical Analysis and Computer Programming	4	40+10	4
	MMATCCT204	Mathematical Methods (Integral Transforms, Integral Equations & Generalized Functions)	4	40+10	4
	MMATCCT205	Operations Research	4	40+10	4
	MMATCCT206	Differential Geometry	4	40+10	4

<b>SEMESTER-III</b>	<b>Code</b>	<b>Course Title</b>	<b>Credit</b>	<b>Marks</b>	<b>No. of Class Hours /Week</b>
	MMATCCT301	Measure and Integrations / Continuum Mechanics and Electromagnetic theory	4	40+10	4
	MMATMET302	Spl. Paper 1	4	40+10	4
	MMATMET303	Spl. Paper 2	4	40+10	4
	MMATOET304	Statistical Methods	4	50	4
	MMATCCS305	Computer aided Numerical Practical	4	50	8
	MMATOPP306	Outreach Programme	4	50	8

	<b>Code</b>	<b>Course Title</b>	<b>Credit</b>	<b>Marks</b>	<b>No. of Class Hours /Week</b>
<b>SEMESTER-IV</b>	MMATCCT401	Graph Theory and Mathematical Logic	4	40+10	4
	MMATCCT402	Operator theory and Elements of Dynamical System	4	40+10	4
	MMATMET403	Spl. Paper 3	4	40+10	4
	MMATMET404	Spl. Paper 4	4	40+10	4
	MMATACT405	Computer Application	4	50	4
	MMATMEP406	Project	4	50	8

<b>Cluster of Special paper 1 and 2 for Semester III</b>	<b>Cluster of Special paper 3 and 4 for Semester IV</b>
<b>A1: Advanced Complex Analysis I</b>	<b>B1: Advanced Complex Analysis II</b>
<b>A2: Advanced Functional Analysis I</b>	<b>B2: Advanced Functional Analysis II</b>
<b>A3: Advanced Real Analysis I</b>	<b>B3: Advanced Real Analysis II</b>
<b>A4: Advanced Algebraic Topology I</b>	<b>B4: Advanced Algebraic Topology II</b>
<b>A5: Advanced Differential Geometry of Manifolds-I</b>	<b>B5: Advanced Differential Geometry of Manifolds-II</b>
<b>A6: Advanced Computational Fluid Dynamics I</b>	<b>B6: Advanced Computational Fluid Dynamics II</b>
<b>A7: Advanced Fluid Mechanics I</b>	<b>B7: Advanced Fluid Mechanics II</b>
<b>A8: Advanced Viscous Flows, Boundary Layer Theory and Magneto-hydrodynamics I</b>	<b>B8: Advanced Viscous Flows, Boundary Layer Theory and Magneto-hydrodynamics II</b>
<b>A9: Advanced Operations Research I</b>	<b>B9: Advanced Operations Research II</b>
<b>A10: Advanced Quantum Mechanics I</b>	<b>B10: Advanced Quantum Mechanics II</b>
<b>A11: Advanced Theory of Elasticity I</b>	<b>B11: Advanced Theory of Elasticity II</b>
<b>A12: Advanced Topology I</b>	<b>B12: Advanced Topology II</b>
<b>A13: Advanced Algebra I</b>	<b>B13: Advanced Algebra I</b>
<b>A14: Advanced Computational Mathematics I</b>	<b>B14: Advanced Computational Mathematics II</b>

### Detailed Syllabus

<b>SEMESTER-I</b>
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#### **MMATCCT101: Abstract Algebra (Marks: 50)**

**Group Theory:** Group isomorphism theorems, automorphism groups, Inner automorphisms, Normal subgroups and correspondence theorem for groups, Direct product (internal and external), Group action on a set, Conjugacy classes and class equation, p-groups, Cauchy's theorem, converse of Lagrange's theorem for finite commutative groups, Sylow theorems and their applications, Normal and subnormal series, composition series, solvable series, solvable and Nilpotent groups, Jórdan-Holder Theorem, Finitely generated Abelian groups(statement only), Free Abelian groups.

**Rings :** Quotient field of an integral domain, Isomorphism theorems for rings, Prime ideal, maximal ideal, primary ideal, Euclidean domain, Principal ideal domain, Unique factorization domain, Gauss' theorem, Polynomial rings, Rings with chain conditions-Noetherian and Artinian Rings; Semi Simple Ring, Jacobson's radical, Hilbert basis theorem.

**Field Extensions :** Field extension – algebraic and transcendental extension and their characterizations. Splitting field, algebraic closure and algebraically closed field. Separable and normal extension. Galois field.

#### **References**

1. Herstein, I. N., Topics in Algebra (Vikas).
2. Dammit & Fote, Abstract algebra
3. Hungerford, T. W., Algebra (Springer).
4. Malik, Mordeson & Sen, Fundamentals of Abstract Algebra (Tata MaGraw-Hill)
5. Sen, Ghosh & Mukhopadhyay, Topics in Abstract Algebra (University Press).
6. Cohn, P. M., Basic Algebra.
7. Lang, S., Algebra.
8. Fraleigh, J.B., A First Course in Abstract Algebra
9. Gallian, J. A., Contemporary Abstract Algebra

#### **MMATCCT102: Real Analysis (Marks: 50)**

Functions of bounded variation on an interval, their properties, Rectifiable Curves. Riemann-Stieltjes integral. Change of variable in a Riemann-Stieltjes integral, Reduction to Riemann integral, Riemann-Stieltjes integral when integrator is of bounded variation, necessary as well as sufficient conditions for existence of Riemann-Stieltjes integrals, Lebesgue's criterion for existence of Riemann integrals.

Differentiation on  $\mathbb{R}^n$  : Directional derivatives, Directional derivatives and continuity, Total derivative and continuity, expression of total derivative in terms of partial derivatives, the matrix transformation of  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ . The Jacobian matrix.

Matrix form of Chain rule. Mean value theorem for vector valued functions. A sufficient condition for differentiability. A sufficient condition for equality of mixed partial derivatives. Taylor's Formula for functions from  $\mathbb{R}^n \rightarrow \mathbb{R}$ ; Functions with non-zero Jacobian determinant, the inverse function theorem, the implicit function theorem as an application of Inverse function theorem.

Extrema of Real-valued functions of several variables; Extremum problems with side conditions – Lagrange's necessary conditions as an application of Inverse function theorem.

Riemann integral of a bounded function defined on a compact interval in  $\mathbb{R}^n$ . Evaluation of multiple integral by iterated integration, Mean value theorem for multiple integral.

### References

1. Natanson, I. P., Theory of Functions of a Real Variable, Vol. I
2. Goffman, C., Real Functions
3. Burkil & Burkil, Theory of Functions of a Real Variable
4. Goldberg, Real Analysis
5. Royden, Real Analysis
6. Lahiri & Roy, Theory of Functions of a Real Variable
7. Apostol, Mathematical Analysis
8. Titchmarsh, Theory of Functions
9. Scwarz, C., Measure, Integration and Functions Spaces.
10. Rudin, Principles of Mathematical Analysis.

### MMATCCT103: Complex Analysis (Marks: 50)

Riemann's sphere, point at infinity and the extended complex plane.

Functions of a complex variable, continuity and Differentiability. Analytic functions, Cauchy-Riemann equations, Branch of a logarithm and multivalued function, Harmonic functions, The Milne-Thompson Method.

Power Series: Infinite series, uniform convergence, Behaviour of Power series on a circle of convergence, Analiticity of Power Series.

Maximum and Minimum Modulus theorem. Some consequences of Maximum Modulus theorem. Open Mapping Theorem.

Complex Integration: Curves and Contours, Simply connected region, Complex Integration, Cauchy-Goursat theorem, Cauchy's integral formula. Liouville's theorem. Fundamental theorem of algebra., Morera's theorem. Idea of winding number.

Taylor Series, Laurent's series and classification of singularities: Taylor and Laurent's series, classification singularities. Casorati-Weierstrass's theorem. Cauchy's Residue theorem and evaluation of certain integrals.

Meromorphic Functions: Introduction. Argument principle, Rouché's theorem its application.

Conformal mapping, Bilinear transformation. Idea of analytic continuation.

### References

1. Markushevich, A. I., Theory of Functions of a Complex Variable( Vol. I, II and III).
2. Churchill, R. V. and Brown, J. W., Complex Variables and Applications.
3. Titchmarsh, E. C., The Theory of Functions.

4. Copson, E. T., An Introduction to the Theory of Functions of a Complex Variable.
5. Conway, J. B., Functions of One Complex Variable.
6. Ahlfors, L. V., Complex Analysis.
7. Punoswamy, Functions of Complex Variable
8. Ganguly S. and Mondal, D., Lecture course on Complex Analysis, Academic Publishers

### **MMATCCT104: Ordinary & Partial Differential Equations (Marks: 50)**

#### **Ordinary Differential Equations**

Adjoint and self-adjoint equations, Green's function and its properties, Green's functions for ordinary differential equations and its application to boundary value problems.

Ordinary point and singularity of a second order linear differential equation in the complex plane, Fuch's theorem, Series Solution about an ordinary point, Regular singularity, Frobenius' method, Series Solution about a regular singularity.

Hypergeometric equation, Hypergeometric functions, Series solution near zero, one and infinity. Integral formula for the hypergeometric function, Differentiation of hypergeometric function.

Legendre polynomial and its generating function, Rodrigue's formula, Recurrence relations and differential equations satisfied by it, Its orthogonality, Expansion of a function in a series of Legendre Polynomials, Legendre functions of first kind and second kind.

Bessel's equation, Bessel's function, Series solution of Bessel's equation, Generating function, Integral representation of Bessel's function, Recurrence relations.

#### **Partial Differential Equations**

Classification and reduction of a second order linear PDE to normal form, Solutions of equation with constant coefficients, Solutions of nonlinear equations of second order by Monge's method.

Dirichlet's and Neumann's interior and exterior problems.

The equation of vibration of a string, Mixed initial and boundary value problem, Existence, uniqueness and continuous dependence of the solution, D'Alembert's solution, Generalized solution, Domain of dependence and domain of influence, Method of separation of variables for the solution of the problem of a vibrating string. Riemann's method of solution.

Laplace's equation, Fundamental solutions of Laplace's equation in two and three dimensions, Minimum-maximum theorem and its consequences, Uniquenesstheorem, Mean value theorem, Boundary value problems. Method of separation of variables for the solution of Laplace's equations in two or three dimensions. Green's function for the Laplace equation in two and three dimensions, Dirichlet's principle, Rayleigh-Ritz method.

Heat equation in two independent variables, First boundary value problem, Maximum-Minimum theorem and its consequences, Continuous dependence of the solution and existence of it, Uniqueness and stability of solution, solution of Cauchy problem using Dirac-Delta function.

### References

1. Codrington, E.A. and Leinson, N., Theory of Ordinary Differential Equation, McGraw-Hill.
2. Simmons, G.F., Differential Equations, Tata McGraw Hill
3. Ross, S.L., Ordinary Differential Equations, John Wiley & Sons
4. Sneddon., I.N. , Special Functions of Mathematical Physics and Chemistry, Oliver and Boyd, London.
5. Rainville, E.D., Special Functions, Macmillan.
6. Lebedev, N.N., Special Functions and Their Applications.
7. Burkhill, J.C., Theory of Ordinary Differential Equation.
8. Ince, E.L., Ordinary Differential Equation, Dover.
9. Sneddon, I.N., Elements of Partial Differential Equations, McGraw Hill.
10. Miller, Partial Differential Equations. John, F., Partial Differential Equations.
11. Amarnath, T., An Elementary Course in Partial Differential Equations, Narosa Pub.
12. Prasad, P., Ravindran R., Partial Differential Equations, New Age International (p) Ltd.
13. Williams, W.E., Partial Differential Equations.
14. Petrovsky, I.G., Lectures on Partial differential equations.
15. Courant & Hilbert., Methods of Mathematical Physics, Vol-I, II.

### **MMATCCT105: Classical Mechanics & Calculus of Variations (Marks: 50)**

Generalised co-ordinates, Constraints and the forces responsible for it, Types of Constraints, Degrees of freedom, Dynamical system and its classification, Virtual Work, D'Alembert principle, Generalised forces and generalised momentum, Kinetic energy, Lagrange's equation of motion (first kind),

Lagrange's equations of motion (second kind) for holonomic and non-holonomic systems, Lagrange's equations of motion (second kind) for velocity dependent potential field, Dissipative forces and dissipation function. Configuration space and system points, Action Integral; Hamilton's principle; Lagrange's equations by variational methods; Hamilton's principle for non-holonomic system; Symmetry properties and conservation laws; Noether's theorem. Point transformations. Cyclic co-ordinates, Routh's process for the ignoration of co-ordinates and its applications, Legendre dual transformation, Hamilton's canonical equations of motion, Principle of least action, Principle of energy.

Generating functions, Canonical transformations and its properties, Condition for canonicity, Infinitesimal canonical transformations, Lagrange and Poisson brackets and their properties, Invariance of Poisson and Lagrange brackets, Representation of Hamilton's equations of motion in terms of Poisson bracket, Jacobi's identity. Hamilton Jacobi Theory.

Motion of a rigid body rotating about a fixed point, Expressions for velocity, angular momentum and kinetic energy, Euler's dynamical equations and their solutions, Euler angles, Angular velocity in terms of Euler angles, Motion of a top in a perfectly rough floor and its stability, Corioli's force.



**Calculus of Variations:** Variation, Linear functional, Deduction of Euler-Lagrange differential equation and some special cases of it, Euler-Lagrange differential equation for multiple dependent variables, Functional dependent on higher order derivatives, Functional dependent on functions of several variables. Application of Calculus of variations for the problems of shortest distance, minimum surface of revolution, Brachistochrone problem, geodesic, Isoperimetric problem, Calculus of variations for problems in parametric form, Variational problems with moving boundaries.

### References

1. Goldstein. H, Classical Mechanics, Narosa Publ., New Delhi, 1998.
2. Rana and Joag, Classical Mechanics, Tata McGraw Hill, New Delhi, 2002.
3. Gantmacher. F, Lectures in Analytical Mechanics, Mir Publ., 1975.
4. Kibble and Berkshire, Classical Mechanics, 4<sup>th</sup> ed., Addison-Wesley Longman, 1996.
5. Chetaev.N.G, Theoretical Mechanics, Springer-Verlag, 1990.
6. Calkin. M, Lagrangian and Hamiltonian Mechanics, World Sci. Publ., Singapore, 1996.
7. Synge and Griffith, Principles of Mechanics, McGraw Hill, Singapore, 1970.
8. Taylor. J.R, Classical Mechanics, University Science Books, California, 2005.
9. Landau and Lifshitz, Mechanics, 3<sup>rd</sup> ed., Pergamon Press, 1982.
10. Chorlton, F., Dynamics.
11. Synge and Graffith, Principle of Mechanics, Mc. Graw-Hill Book Co. 1960 .
12. Green Wood, D.T., Classical Dynamics, Dover Publication, 2006.
13. Gupta, K. C., Classical Mechanics of Particles and Rigid Bodies, John Wiley & Sons Inc., 1988.
14. Torok, Analytical Mechanics.
15. Gupta, A. S., Calculus of Variations with Applications, Prentice –Hall of India, 1996.
16. Gelfand, I. M. and Fomin, S.V., Calculus of Variations, Prentice Hall Inc, 2012.
17. Elsgots, Calculus of Variations, Mir Publ.

### MMATCCT106: Topology ( Marks: 50)

#### Fundamentals of Topological spaces:

Topological spaces, open and closed sets. Bases and sub-bases. Closure & interior -their properties and relations; exterior, boundary, accumulation points, derived sets, dense set,  $G_\delta$  and  $F_\sigma$  sets. Neighbourhood system. Order Topology.

Subspace and induced / relative topology. Relation of closure, interior, accumulation points etc., between the whole space and the subspace.

Alternative way of defining a topology using Kuratowski closure operator, interior operator and neighbourhood systems; Continuous Functions, Open maps, Closed maps and Homeomorphisms.

**Countability axioms:** 1<sup>st</sup> and 2<sup>nd</sup> countability axioms, Separability and Lindeloffness. Characterizations of accumulation points, closed sets, open sets in a 1<sup>st</sup> countable space w.r.t. sequences. Heine's continuity criterion.

**Separation Axioms:**  $T_i$  spaces ( $i = 0, 1, 2, 3, 3\frac{1}{2}, 4, 5$ ), their characterizations and basic properties. Urysohn's lemma and Tietze's extension theorem (statement only) and their applications.

**Compactness:** Compactness and its basic properties, Alexander subbase theorem, Continuous functions and compact sets. Compactness of  $\mathbb{R}$ . Sequential compactness, BW Compactness and countable compactness. Lebesgue Number.

**Connectedness:** Connected and disconnected spaces, Path Connected Spaces, Connected Sets in  $\mathbb{R}$ ,  $\mathbb{R}^n$  ( $n > 1$ ), Components and Path Components, quasi-components.

**Product spaces:** Product and box topology, Projection maps. Tychonoff product theorem. Separation axioms, Countability axioms and Connectedness in product spaces.

### References

1. Kelley, J. L. and Nostrand, V., General Topology
2. Willard, S., General Topology, Addison-Wesley
3. Dugundji, J., Allyn and Bacon, Topology
4. Munkres, J., Topology, A first course, Prentice Hall, India
5. Simmons, G. F., Introduction to topology and modern analysis, McGraw Hill
6. Joshi, K. D., Introduction to General topology, Wiley Eastern Ltd.
7. Engelking, General Topology, Polish Scientific Publishers, Warszawa
8. Steen L. and Seebach, J., Counter examples in Topology
9. Chatterjee, B. C., Ganguly S. and Adhikari, M., A text book of Topology, Asian Books
10. Thron, W. J., Topological Structures

**SEMESTER-II**

**MMATCCT201: Linear Algebra (Marks: 50)**

Linear transformations and its matrix representation, rank and nullity, annihilator of a subset of a vector space. Linear Functionals, Dual Spaces, Dual Basis. Eigen vectors and eigen values, similar and congruent matrices, characteristic polynomial, minimal polynomial, Determinant divisors and invariant factors, description of minimal polynomial in terms of invariant factors,

Reduction to Triangular Forms, Diagonalization, diagonalization of symmetric and Hermitian matrices. Jordan Blocks and Jordan Canonical Forms, Invariant Factors, Rational Canonical Forms, Smith Normal Form.

Bilinear Forms, Quadratic Forms, Hermitian Forms, Positive Definite Forms & Matrices, Principal Minor Criterion, Direct Sum Decomposition, Signature, Sylvester Law Of Inertia, Simultaneous Reduction of Pair of Forms, applications to Geometry & Mechanics.

Modules over rings, Submodules, Quotient modules, Free modules, Abelian groups as modules over the ring of integers, Structure theorem for finitely generated abelian groups.

**References**

1. Lang, S., Linear Algebra.
2. Hoffman & Kunze, Linear Algebra (Prentice Hall).
3. Kumareson, S., Linear Algebra.
4. Rao & Bhimsankaran, Linear algebra.
5. Friedberg & Spence, Linear Algebra
6. Curtis, Linear Algebra
7. I.A., Herstein, Topics in Algebra
8. S. K. Berberian, Linear Algebra
9. M. Artin, Algebra
10. Dummit and Foote, Abstract Algebra, 2010
11. T.S. Blyth, Module Theory

**MMATCCT202: Functional Analysis (Marks: 50)**

Review of continuity, completeness and compactness in metric spaces. Hölder and Minkowski inequalities (statement only). Baire's (category) theorem, Banach's fixed point theorem, applications to solutions of certain systems of linear algebraic equations, Fredholm's integral equation of the second kind.

Normed linear spaces, Banach spaces and their examples; completion of normed space; finite dimensional normed space and its properties, equivalent norms, compactness of normed space, Riesz lemma and its application,

Bounded linear operators and its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Hahn-Banach theorem and some of its applications.

Dual spaces, Examples, Separability of the Dual space, Reflexive spaces, Examples, Uniform boundedness principle and its simple applications, The Open mapping Theorem & the Closed graph Theorem.

Inner product space, Hilbert space, examples, completion of inner product space, parallelogram law, polarization identity, C-S inequality, Pythagorean theorem, orthogonal complement, direct sum, Projection theorem, orthonormal sets and its properties, Bessel's inequality, Gram Schmidt orthogonalization process, complete orthonormal sets, Parseval identity. Riesz- Fischer Theorem. The Riesz representation theorem.

### References

1. Lusternik and Sovolev, Functional Analysis.
2. Siddiqui, A.H. Functional Analysis with applications, TMG Publishing Co. Ltd, New Delhi.
3. Jha, K.K., Functional Analysis, Student's Friends, 1986.
4. Vulikh, Functional Analysis.
5. Bachman, G. & Narici, L., Functional Analysis, Academic Press, 1966.
6. Taylor, A.E., Functional Analysis, John Wiley and Sons, New York, 1958
7. B.V. Limaya, Functional Analysis

## **MMATCCT203: Numerical Analysis and Computer Programming in C (Marks: 50)**

### **Group-A: Numerical Analysis (Marks: 30)**

Polynomial interpolation, Errors and minimizing errors, Hermite's interpolation, Piecewise polynomial approximation, Cubic spline interpolation, Approximations with orthogonal polynomials, Chebyshev polynomials.

Richardson extrapolation and Romberg's integration, Gaussian quadrature, Gauss-Legendre and Gauss-Chebyshev integration rule, Quadrature formula for singular integrals.

QD algorithm and Bairstow's method.

Solution of second order ordinary differential equations and simultaneous first order ordinary differential equations using fourth order R-K method. Adam-Bashforth-Moulton and Milne's predictor-corrector method for the solution of initial value problems.

Solution of second order boundary value problems by finite difference method and Shooting method.

Determination of extreme eigenvalues and related eigenvectors by Power method, Jacobi's method for all eigenvalues.

Finite difference approximations to partial derivatives, Schmidt explicit and Crank-Nicolson implicit method for the solution of parabolic equations in one space co-ordinate, Implicit finite difference method for solution of Hyperbolic equation in one space co-ordinate, Solution of elliptic equation in two variables, convergence and stability analysis.

### References

1. Hildebrand, F. B., Introduction to Numerical Analysis.
2. Isaacson and Keller, Analysis of Numerical methods.
3. Jain, M.K., Numerical solution of differential equations.
4. Atkinson, Numerical Analysis, John Wiley & Sons, Singapore, 1989.
5. Gupta, A and Basu, S.C., Numerical Analysis.
6. Jain, Iyenger and Jain, Numerical Methods for Scientific and Engineering Computation, 4th Ed, New Age International (P) Ltd., New Delhi, 2003 .
7. Sastry, S.S., Introductory methods of Numerical Analysis, Prentice Hall India Pvt. Ltd., New Delhi, 1999.

### Group-B: Computer Programming in C (Marks: 20)

**Introduction:** Basic structures, Character set, Keywords, Identifiers, Constants, Variable-type declaration

**Operators:** Arithmetic, Relational, Logical, assignment, Increment, decrement, Conditional. Operator precedence and associativity, Arithmetic expression,

**Statement:** Input and Output, Define, Assignment, User define, Decision making (branching and looping) – Simple and nested IF, IF – ELSE, LADDER, SWITCH, GOTO, DO, WHILE – DO, FOR, BREAK AND CONTINUE Statements. Arrays- one and two dimensions, user defined functions,

**String Manipulation functions:** String Manipulation functions in C, Operations and characters.

**Pointer:** Declarations, Address operator, pointer as functions-call by value, call by reference, pointer arithmetic.

### References

1. Xavier, C., C Language and Numerical Methods, (New Age Intl (P) Ltd. Pub.)
2. Gottfried, B. S., Programming with C (TMH).
3. Balaguruswamy, E., Programming in ANSI C (TMH).
4. Scheid, F., Computers and Programming (Schaum's series)
5. Jeyapooan, T., A first course in Programming with C.
6. Litvin and Litvin, Programming in C++.

**MMATCCT204: Mathematical Methods (Marks: 50)**

**(Integral Transforms, Integral Equations & Generalized Functions)**

**Integral Transforms (20 marks)**

Fourier integral theorem, Definition of Fourier Transforms, Algebraic and analytic properties of Fourier Transform, Fourier sine and cosine Transforms, Fourier Transforms of derivatives, Fourier Transforms of some useful functions, Inversion formula of Fourier Transforms, Convolution Theorem, Parseval's relation, Applications of Fourier transforms in solving ordinary and partial differential equations.

Definition and properties of Laplace transforms, Sufficient conditions for the existence of Laplace Transform, Convolution theorems, Inverse of Laplace Transform, Evaluation of inverse Laplace Transform by residue calculation, Bromwich integral, Application to Ordinary and Partial differential equations.

**Integral Equations (20 marks)**

Linear integral equations of first and second kinds, Fredholm and Volterra types integral equations, Reduction of boundary value problem of an ordinary differential equation to an integral equation and vice-versa.

Existence and uniqueness of solutions of Fredholm and Volterra's integral equations of second kind, Solution by the method of successive approximations, Neumann series, Solution by Resolvent kernel method and iterated kernel method.

Integral equations with degenerate kernels, Fredholm theorem, Fredholm alternatives, Eigenvalue and eigenfunction of integral equation and their elementary properties.

Integral equations with symmetric kernels, Properties of symmetric kernels, Integral equations of convolution type and their solutions by Laplace transform.

Singular integral equations, Abel integral equation and its solution.

**Generalized Functions (10 marks)**

Basic concepts of Linear functional and test functions; regular and singular distributions, differentiation of a distribution; Sequence and series of distributions, Convergence of distributions, Dirac Delta function, its transformation properties, Generalized Fourier series.

**References**

1. Sneddon, I.N., Fourier Transforms, McGraw-Hill Pub, 1995.
2. Sneddon, I.N., Use of Integral Transforms, McGraw-Hill Pub.
3. Andrews, L.C., Shivamoggi, B., Integral Transforms for Engineers, PHI.
4. Debnath, L., Bhatta, D., Integral Transforms and Their Applications, CRC Press, 2007.
5. Tricomi, F.G., Integral Equation, Interscience Publishers, 1985.

6. Mikhlin, S.G., Integral Equation, Pergamon Press, 1960.
7. Chakraborti, A., Applied Integral Equation, Vijay Nicole Imprints Pvt Ltd.
8. Lovit, W.V., Linear Integral Equations, Dover Publishers, 2005.
9. Stackgold I., Green's Functions and Boundary Value Problems, *John Wiley & Sons, New York* (1979).
10. Gorain, G. C., Laplace Transformation, Narosa Publishing, 2014.

### **MMATCCT205: Operations Research (Marks: 50)**

**Introduction:** Definition of O.R., Drawbacks in definition, Scope of O.R., O.R. and decision making, Application of O.R. in different sectors, Computer application in O.R.

**Revised simplex method:** Standard forms of revised simplex method, Computational procedure, Comparison with simplex method.

Sensitivity Analysis.

Bounded variable method. Dual Simplex method.

**Integer Programming:** Introduction, Cutting plane method, Branch and bound technique, Binary linear programming, Travelling salesman problem.

**Inventory control Models:** Classification of Inventories, Advantage of Carrying Inventory, Features of Inventory System, Deterministic inventory models including price breaks, probabilistic inventory models.

**Job sequence:** Processing of  $n$  jobs through two machines, The Algorithm, Processing of  $n$  jobs through  $m$  machines

**Project Network:** Introduction, Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Critical Path analysis, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project.

### **References**

1. Wagner – Principles of Operations Research (PH)
2. Sasievir, Yaspan, Friedman – Operations Research: Methods and Problems (JW)
3. Sharma J K – Operations Research – Theory and Applications
4. Hillie & Lieberman – Introduction to Operations Research
5. Swarup, Gupta & Manmohan – Operations Research
6. Kapoor V.K. -- Operations Research
7. Hadley G., -Linear Programming, Narosa Publishers, 1987
8. Hillier & Lieberman—Introduction to Operations Research, 7/e, TMH

**MMATCCT206: Differential Geometry (Marks: 50)**

**Geometry of  $\mathbf{R}^n$**  : Hyperplanes in  $\mathbf{R}^n$ , Lines and planes in  $\mathbf{R}^n$ , Parametric equations, Inner product in  $\mathbf{R}^n$ , Orthonormal basis, Orthogonal transformations, Orthogonal matrices, The groups  $SO(2)$ ,  $SO(3)$ , Reflections and rotations, Isometries of  $\mathbf{R}^n$ .

**Curves in Space:** Parametric representation of curves, Helix, Curvilinear coordinates in  $E^3$ . Tangent and first curvature vector, Serret- Frenet formulas for curves in space, Frenet formulas for curve in  $E^n$ . Intrinsic differentiation, Parallel vector fields, Geodesic.

**Surfaces:** Regular surfaces in  $\mathbf{R}^3$ . Tangent space to a surface at a point, Equivalent definitions, Smooth functions on a surface, Differential of a smooth function defined on a surface, Orientable surfaces. Parametric representation of a surface, Tangent and Normal vector field on a surface, The first and second fundamental form of surface, Geodesic curvature of a surface curve, The third fundamental form, Gaussian curvature, Isometry of surfaces, Developable surfaces, Weingarten formula, Equation of Gauss and Codazzi, Principal curvature, Normal curvature, Meusnier's theorem.

**References**

1. Weatherburn, C. E., Differential Geometry
2. Postnikov, M., Lectures in Geometry, Linear Algebra and Differential Geometry
3. De, U. C., Differential Geometry of Curves and Surfaces in  $E^3$ , Anamaya Publi., 2007.
4. Do Carmo, M. P., Differential Geometry of Curves and Surfaces
5. O'Neill, B., Elementary Differential Geometry
6. Rutter, Geometry of Curves
7. M. DoCarmo, Differential geometry of curves and surfaces, Princeton University Press, 1976.
8. S. Montiel and A. Ros, Curves and Surfaces, AMS Graduate Studies in Mathematics, 2009.
9. A. Pressley, Elementary Differential Geometry, Springer UTM, 2009.
10. J. Thorpe, Elementary Topics in Differential Geometry, Springer UTM, 2007.



**SEMESTER-III**

**MMATCCT301: Measure and Integrations / Continuum Mechanics & Electromagnetic Theory  
(Marks: 50)**

**Measure and Integrations (Marks: 50)**

Outer Lebesgue Measure in  $E^*$  (starting with the concept of length of an interval); the properties of outer Lebesgue Measure  $m^*$ . Outer measure  $\mu^*$  on  $S$ -where  $S$  is a space; the concept of  $\mu$ -measurable sets with the help of  $\mu^*$ . Necessary and sufficient condition for  $\mu$ -measurability. Properties of  $\mu$ -measurable sets. The structure of  $\mu$ -measurable sets-the concept of  $\sigma$ -algebra; the  $\sigma$ -algebra of Lebesgue measurable sets.

Properties of Lebesgue measure, Vitali's theorem: The existence of a non-measurable set. The Borel sets & Lebesgue measurable sets- a comparison.

$\mu$ -measurable functions, their properties, characteristic functions; step functions.

Lebesgue Integration. Lebesgue's monotone convergence theorem; Fatou's lemma; the theorem on Dominated summability; Lebesgue's dominated convergence Theorem.

Necessary & Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration.

The Concept of  $L^p$ -spaces; Inequalities of Holder and Minkowski; Completion of  $L^p$ -spaces. Convergence in Measure, Almost Uniform Convergence, Pointwise Convergence a.e.; Convergence Diagrams, Counter Examples. Egoroff theorem.

Lebesgue Integral in the Plane. Product  $\sigma$ -algebra. Product Measure. Fubini's Theorem.

**References:**

1. P. R. Halmos, Measure Theory, Von Nostrand, New York, 1950.
2. E. Hewitt & K. Stromberg, Real and abstract Analysis, Third edition, Springer-
3. G.D. Barra, Measure Theory & Integration, Wiley Eastern Limited, 1987.  
verlag, Heidelberg & New York, 1975.
4. W. Rudin, Real and Complex Analysis, Tata McGraw- Hill, New York, 1987
5. I. K. Rana, An introduction to Measure & Integration, Narosa Publishing House, 1997.
6. H. L. Royden, Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.
7. J. F. Randolph, Basic Real and Abstract Analysis, Academic Press, New York, 1968.
8. C. D. Aliprantis and Owen Burkinshaw, Principles of Real Analysis, Academic Press, 2000.
9. K. R. Parthasarathy, introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
10. R.B. Bartle, Elements of Real Analysis

OR

**Continuum Mechanics & Electromagnetic Theory (Marks: 50)**

**Group A: Continuum Mechanics (Marks: 30)**

Continuous media, Deformable bodies, Body and surface forces, Cauchy's stress principle, Stress tensor, Equation of equilibrium, Symmetry of stress tensor, Principal stresses and principal direction of stresses, Stress invariants, Stress quadric of Cauchy, Maximum normal and shearing Stresses.

Deformation, Deformation Gradients, Finite strain tensor, Infinitesimal strain components, Geometrical interpretation of infinitesimal strain components, Principal strain and principal axes of strain, Compatibility of strain components.

Lagrangian and Eulerian approaches to study the motion of continua, Material derivative of a volume integral, Equation of continuity, Equation of motion, Equation of angular momentum, Energy equation.

Generalized Hooke's law, Elastic Moduli, Equation of motion and equilibrium in terms of displacement components, Beltrami-Michell compatibility equations, Strain energy density function.

Inviscid fluid, Circulation, Kelvin's Energy Theorem, Constitutive equations for viscous fluid, Navier-Stokes equations of motion, Boundary conditions.

**References**

1. Chung, T. J., Continuum Mechanics, Prentice-Hall.
2. Chatterjee, R. N., Mathematical Theory of Continuum Mechanics, Narosa Publishing House, New Delhi, 1999.
3. Fung, Y.C., A first course in continuum mechanics.
4. Eringen, A.C., Non-linear Theory of Continuous Media, McGraw-Hill.
5. Chorlton, F., A text book of Fluid Mechanics.
6. Sedov, L.I., A course in continuum mechanics, Vol-I.
7. Leigh, D.C., Non-linear Continuum Mechanics, McGraw-Hill.
8. Prager, W., Mechanics of continuous media.
9. Sokolnikoff, I.S., Mathematical theory of Elasticity, Tata Mc Grow Hill Co., 1977
10. Landau and Lifshitz, Mechanics, 3<sup>rd</sup> ed., Pergamon Press, 1982

**Group B: Electromagnetic Theory (Marks: 20)**

Electrostatic field, Coulomb's law, Principle of superposition, Boundary conditions; Electrostatic potential, Gauss's law, Poisson equation, Energy in electrostatic field, Electric dipole, Conductor and insulator; Electrostatics in dielectric media, Electric polarization, Energy in dielectric media.

Electric current, Equation of continuity, Ohm's law, Magnetostatic field, Lorentz force, Ampere's law, Biot-Savart's law, Boundary conditions; Magnetic vector potential, Multi-pole expansion, Magnetic dipole; Magnetic field in matter, Magnetization, Auxiliary field.

Electromagnetic induction, Faraday's law; inductance; energy in magnetic field.

Ampere-Maxwell equation; Maxwell's equations in vacuum and in matter, Physical significance, Boundary conditions; Energy transfer and Poynting theorem, Gauge transformations, Coulomb and Lorentz gauges.

### References

1. Griffiths D. J., Introduction to electrodynamics (3<sup>rd</sup> Edition), PHI Learning Private Limited, New Delhi (2012).
2. Coulson A. A., Electricity, Oliver and Boyd, Edinberg & London (1974).

## MMATMET302 & MMATMET303

All the students will have to take two Special Papers in Semester III from the clusters of special papers as given below. The allocation of special papers will be monitored by the Department.

### A1: Advanced Complex Analysis I (Marks: 50)

Convex function and Hadamard three-circle theorem, Phragmen-Lindelof theorem.

Harmonic function, Mean value property, Maximum principle for Harmonic function, Poisson Integral formula- for analytic and for Harmonic functions formula. Theorem of Borel –Caratheodary. Boundary behavior of Poisson Integral. The Phragmen –Lindelof Method. Harnack's inequality & Harnack's Principle,

Conformal Mapping: preservation of angles, Linear fractional transformations, Normal families, The Riemann mapping Theorem. The class  $S$  of analytic and univalent functions on the Unit disk, Kobe one-quarter Theorem. Continuity at the boundary, Conformal Mapping of an Annulus.

Zeros of Holomorphic Functions: Infinite products, Necessary & Sufficient condition for convergence of Product, M-test for the convergence of Product. The Weierstrass factorization Theorem. Mittag- Leffler Theorem, Interpolation Problem, Jensen's Formula, Zeros of Entire Functions. Blaschke Products, the Muntz-Szasz Theorem.

Analytic Continuation: Regular Points and Singular Points, Continuation along Curves, The Monodromy Theorem, Special Functions: The gamma Function of Euler and the Zeta Function of Riemann. Euler's Product Formula. Functional Equation of Zeta Function, Riemann Hypothesis, Construction of a Modular Function. The Picard Theorem

### References

1. Conway, J. B., Functions of One Complex Variable
2. Ahlfors, L. V., Complex Analysis
3. Rudin, W., Real and Complex Analysis
4. Titchmarsh, E. C., Theory of Functions
5. Copson, E. T., Function of a Complex Variable
6. Boas, R. P., Entire Functions
7. Cartan, H., Analytic Functions
8. Markusevich, A. I., Theory of Functions of a Complex Variables, Vol. I & II.

**A2: Advanced Functional Analysis I (Marks: 50)**  
*(To be done in the next meeting)*

**A3: Advanced Real Analysis I (Marks: 50)**

Upper and lower limits of real function and their properties, Pointwise differentiation of functions of linear intervals. Derivates and derivatives. Measurability of derivates. Differentiation of real functions. Dini derivates and their properties. Monotonicity theorem.

Example of a continuous nowhere differentiable function.

Vitali's covering theorem in one dimension. Differentiability of monotone functions. Absolutely continuous functions and singular functions. Cantor ternary set and Cantor function. Indefinite Lebesgue Integral. Fundamental theorem of integral calculus for Lebesgue integral. Lusin's condition (N).

Characterization of absolutely continuous functions (Banach-Zaricki theorem).

References

1. Hewitt and Stormberg, Real and Abstract Analysis
2. Royden, H. L., Real Analysis
3. Saks, Theory of Integrals
4. Rudin, W., Real and Abstract Analysis
5. Munroe, M. E., Measure and Integration
6. Natanson, I. P., Theory of Functions of a Real Variable, Vols. I & II.
7. Hobson, E. W., Theory of Functions of a Real Variable, Vols. I & II

**A4: Algebraic Topology I (Marks: 50)**

Homotopy: Definition and examples of homotopies; Retracts and Deformation retracts; homotopy types and homotopy equivalent spaces.

Path homotopy. Fundamental group, Simply connected spaces and covering spaces, Path lifting and homotopy lifting lemma, Fundamental group of the circle. Fundamental group of a product space.

Fundamental group of  $S^n$ ; Fundamental group of the projective space, Brower fixed point theorem, The Monodromy theorem, Fundamental theorem of algebra, Borsuk-Ulam theorem.

Free abelian groups, Free groups, Free product of groups, Seifert-van Kampen Theorem, Fundamental group of wedge of circles, Fundamental group of the torus.

Equivalence of covering spaces, The lifting lemma, Universal covering space, Covering transformations and group actions, The classification of covering spaces

**References**

1. Massey, W. S., Algebraic Topology

2. Massey, W. S., Singular Homology Theory
3. Spanier, E. H., Algebraic Topology
4. Gray, B., Homotopy Theory: An Introduction to Algebraic Topology
5. Bredon, C. R., Geometry and Topology
6. James Munkres, Topology, Prentice Hall of India, 1992.
7. Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.
8. John Lee, Introduction to Topological Manifolds, Springer GTM, 2000.

### **A5: Differential Geometry of Manifolds-I (Marks: 50)**

**Manifold:** Topological manifold, Differentiable Manifold, Manifolds with boundary, Differentiable function and Differentiable Mapping, Diffeomorphism, Partial derivatives, Inverse Function theorem, Differentiable Curve, Tangent space, Rank of mapping, critical point, regular point, Immersion, Submersion, Embedding, Differential of map, Vector Field, Integral curves, Lie Bracket,  $f$ -related Vector fields, One parameter group of transformations or flow, Local 1-parameter group of transformations or local flow, Existence theorem for ODEs, Submanifolds and hypersurfaces, Categories and functors, Bump function and partition of unity.

**Lie Groups and Lie Algebras:** Lie Group, General Linear Groups, Left translation, Right translation, Invariant Vector Field, Invariant Differential form, Automorphism, One parameter subgroup of a Lie group, Lie Transformation Group (action of a Lie Group on a Manifold), Exponential Maps. Lie Algebra of Vector Fields, Lie derivatives.

### **References**

1. Foundation of differential Geometry (vol-1): S. Kobayashi & K. Nomizu.
2. An Introduction to Differentiable Manifolds and Riemannian Geometry: W. M. Boothby.
3. Introduction to Differentiable Manifolds: L. Auslander & R. E. Mackenzie.
4. Lectures on Differential Geometry: S. S. Chern, W. H. Chen & K. S. Lam.
5. J.M.Lee- Differential Geometry

### **A6: Advanced Computational Fluid Dynamics- I (Marks: 50)**

Finite difference method and its applications to the model equations of parabolic, hyperbolic, elliptic types, Explicit and implicit schemes, Truncation error, consistency, convergence, stability (Von Neumann stability analysis only) of model equation with appropriate initial and boundary conditions, Thomas algorithm, ADI method for two-dimensional heat conduction problem, Splitting and approximate factorization for two-dimensional Laplace equation, Multi-grid method.

First order wave equation, Upwind scheme, consistency, CFL stability condition, First order hyperbolic system, Hyperbolic conservation laws, Convection-diffusion equation, Stability, Finite - Volume method, Galerkin's method, Convergence and stability of Finite-Volume method, Solution of Dirichlet problem for two-dimensional Laplace equation by finite volume method, Simple applications to the problems of Fluid Dynamics.

### References

1. Niyogi, P., Chakraborty, S. K. and Laha, M. K., Introduction to Computational Fluid Dynamics, Pearson education, Delhi, 2005.
2. Fletcher, C. A. J., Computational Techniques for Fluid Dynamics, Vol-1 and Vol-II, Springer.
3. Peyret, R., and Taylor, T. D., Computational Methods for Fluid Flow, Springer 1983.
4. Thompson, J.F., Warsi, Z.U.A. and C.W. Martin, C.W., Numerical Grid Generation, Foundation and Applications, North Holland 1985
5. Landau, L. D., Lifshitz, E. M., Fluid Mechanics, Trans, Pergamon Press 1989
6. Schlichting, H., Gersten, K., Boundary – Layer Theory, 8<sup>th</sup> Ed., Springer 2000

### A7: Advanced Fluid Mechanics I (Marks: 50)

Lagrange's and Euler's approaches in description of fluid motion, Inviscid incompressible fluid, Euler's equation of motion and its vector invariant form, Equation of continuity, Boundary conditions, Material surface, Stream lines and path lines and streak lines of particles, Irrotational and rotational flows, Velocity potential, Bernoulli's equation and its applications to some special cases, Impulsive motion and its properties.

Theory of irrotational motion, Circulation, Kelvin's circulation theorem, Permanence of irrotational motion, Connectivity of regions of space, Acyclic irrotational motion and its properties, Kinetic energy, Kelvin's minimum energy theorem, Uniqueness theorem.

Two dimensional motion, Stream function, Complex potential, Sources, Sinks, Doublets and their images, Circle theorem, Theorem of Blasius, Motion of circular and elliptic cylinders, Circulation about circular and elliptic cylinder, Steady streaming with circulation, Rotation of elliptic cylinder, Theorem of Kutta and Juokowski, Conformal transformation, Axi-symmetric motion, Stokes' stream function, Three-dimensional motion, Source, sink, doublet in three dimension.

Motion of a sphere, Stoke's stream function, Source, sinks, doublets and their images with regard to a plane and sphere.

Vortex line and filament, Equation of surface formed by streamlines and vortex lines in case of steady motion, Strength of a vortex filament, Velocity field and kinetic energy of a vortex system, Uniqueness theorem, Rectilinear vortices, Vortex pair, Vortex doublet, Image of a vortex with regard to a plane and a circular cylinder, Karman's vortex street.

Surface waves, Paths of particles, Energy of waves, Group velocity, Energy of a long wave.

### References

1. Ramsay, A.S., Hydrodynamics (Bell).
2. Lamb, H., Hydrodynamics (Cambridge)
3. Landau, L.D., Lifchiz, E.M., Fluid Mechanics (Pergamon), 1959
4. Milne-Thomson, I.M., Theoretical Hydrodynamics
5. Chorlton, F., Textbook of Fluid Dynamics.

**A8: Advanced Viscous Flows, Boundary Layer Theory and  
Magneto-hydrodynamics I (Marks - 50)**

Viscous fluids, Stress-strain relations for fluids (statement only), Navier-Stokes' equations in Cartesian Co-ordinates and its equivalent forms in spherical, polar and cylindrical co-ordinates, Dimensionless form of Navier-Stokes' equations, Reynolds number and its physical interpretation.

Exact solutions of Navier-Stokes' equations for flow due to suddenly accelerated plane wall, flow near an oscillating plane wall, two-dimensional stagnation-point flow, Karman's flow and its solution.

Creeping motion, Stokes' flow, Steady motion of a viscous fluid due to a slowly rotating sphere, Steady motion between two parallel planes, Stokes' solution for slow steady parallel flow past a sphere, Oseen's approximation, Oseen's solution for slow steady parallel flow past a sphere.

Concept of boundary layer and Prandtl's assumptions, Derivation of boundary layer equations, Boundary layer parameters, Flow over a flat plate, Blasius equation and approximate solution (in the form of an infinite series) for steady flow past a flat plate, Self-similar solution of boundary layer equations, Boundary layer flow past a wedge, Steady boundary layer flow along the wall of a convergent channel, Jet flows (two dimensional and axi-symmetric jets), Separation of boundary layer flow.

Karman's momentum integral equation, Karman-Pohlhausen method and simple applications.

**References**

1. F. Chorlton, Text Book of Fluid Dynamics, Van Nostrand Reinhold Co., London, 1990.
2. J. L. Bansal, Viscous Fluid Dynamics-2<sup>nd</sup> Edition, Oxford and IBH Publishing Co, 1977.
3. H. Schlichting, Boundary Layer Theory, Springer, 2003.
4. S.W. Yuan , Foundations of Fluid Mechanics, Prentice – Hall International, 1970.
5. P. K. Kundu, Fluid Mechanics, Academic Press, 1990.

**A9: Advanced Operations Research I (Marks: 50)**

**Non-linear Programming:** Local and global minima and maxima, convex functions and their properties, Method of Lagrange multiplier, The Kuhn Tucker conditions. Convex programming.

**Quadratic Programming:** Wolfe's Modified Simplex method, Beale's method.

**Separable convex programming, Separable Programming Algorithm**

**Unconstrained Optimization:** Search method : Fibonacci search, Golden Section search; Gradient : Steepest descent Quasi-Newton's method, Davidon-Fletcher-Powell method, Conjugate direction method (Fletcher-Reeves method).

**Replacement Methods:** Introduction, Replacement policies for items whose efficiency deteriorates with time, Individual and group replacement, Replacement policies for items that fail completely.

**Flow Network:** Max-flow min-cut theorem, Generalized Max flow min-cut theorem, Linear Programming interpretation of Max-flow min-cut theorem, Minimum cost flows, Min-flow max-cut theorem.

### References

1. Sharma J K – Operations Research – Theory and Applications
2. Taha – Operations Research
3. Schaum's Outline Series – Operations Research
4. Hillie & Lieberman – Introduction to Operations Research
5. Swarup, Gupta & Manmohan – Operations Research
6. Kapoor V.K. - Operations Research
7. Hillier & Lieberman—Introduction to Operations Research, 7/e (with CD),TMH
8. Rao S.S. –Optimization theory and Applications, Wiley Eastern Ltd., New Delhi.

### A10: Advanced Quantum Mechanics I (Marks: 50)

**Transformation Theory:** Adjoint operator, Hermitian operator, Projection operator, Degeneracy, Unitary transformation, Matrix representation of wave functions and operators, Change of basis, Transformation of matrix elements, Dirac's Bra and Ket notation, Completeness and normalization of eigen functions, Common set of eigen functions of two operators, Compatibility of observables.

**Symmetries and Invariance:** Angular momentum eigenvalues and eigenfunctions, Spin, Addition of two angular momenta, Rotation groups, Identical particles, Pauli exclusion principle, Invariance and conservation theorems.

**Relativistic Kinematics:** Klein-Gordon equation, Dirac equation for a free particle and its Lorentz covariance, Hole theory and positron, Electron spin and magnetic moment.

Approximation Methods (time-independent)

Rayleigh-Schrödinger perturbation method, An harmonic oscillator, Stark effect in hydrogen atom, Zeeman effect, Ground state energy of helium atom.

**Elements of Second Quantization of A System:** Creation and Annihilation operator, Commutation and Anti-commutation rules, Relation with Statistics - Bosons and Fermions.

### References

1. Messiah, A. – Quantum Mechanics, Vol. I & II (North – Holland Pub. Co., 1962).
2. Bransden, B. H. & Joachain, C. J. – Introduction to Quantum Mechanics (Oxford University Press, 1989).
3. Burke, P. G.– Potential Scattering in Atomic Physics (Plenum Press, New York, 1977)
4. Joachain, C. J. – Quantum Collision Theory (North-Holland Pub. Co., 1975)
5. Bransden, B. H. – Atomic Collision Theory (W. A. Benjamin Inc., N. Y., 1970)
6. Geltman, S. – Topics in Atomic Collision Theory (Academic Press. 1969)
7. Mott, Wu, T. Y. and Olmura, T.– Quantum Theory of Scattering (Prentice Hall, New Jersey, 1962)
8. N. F. & H. S. W. Massey – Theory of Atomic collisions (3rd ed.), (Clarendon Press, Oxford, 1965)
9. Goldberger, M. L. & Watson, K. M. – Collision Theory (Wiley, N. Y., 1964)
10. Newton, R. G. – Scattering Theory of Waves and Particles (McGraw – Hill, 1966)



**A11: Advanced Theory of Elasticity-I (Marks: 50)**

Motion of Deformable Bodies: Lagrangian and Eulerian descriptions, Material derivative, Conservation of mass, Equation of continuity, Momentum principles, Equation of motion, Energy balance.

Constitutive Equations of ideal materials, Classical elasticity, Generalised Hooke's law, Isotropy, Elastic Moduli.

Equations of motion and equilibrium in terms of displacement components, Beltrami-Michell compatibility equations, Strain energy density functions, Saint-Venant's principle, Boundary value problems of Static and dynamic elasticity, Uniqueness of Solutions.

Plane stress, Generalized plane stress, Airy's stress function, General solution of bi-harmonic equation, Stresses and displacements in terms of complex potentials, Simple problems. Stress function approach to problems of plane stress.

**References**

1. Love, A.E.H., A Treatise on the Mathematical Theory of Elasticity, Dover.
2. Sokolnikoff, I.S., Mathematical Theory of Elasticity, McGraw Hill, 1956.
3. Chung, T.J., Continuum Mechanics, Prentice Hall, 1988
4. Fung, Y.C., Foundations of Solid Mechanics.
5. Spencer, A.J.M., Continuum Mechanics, Longman, 1980.

**A12: Advanced Topology- I (Marks: 50)**

Nets and Filters : Inadequacy of sequences. Nets & filters. Topology and convergence of nets & filters. Subnets. Ultranets & Ultra filters. Canonical way of converting nets to filters and vice-versa. Characterizations of Hausdorffness, compactness and continuity in terms of nets and filters. Convergence of nets and filters in product spaces.

Identification topology and Quotient spaces.

Local Connectedness, Path-connectedness, Total disconnectedness, Zero-dimensional spaces, Extremely disconnected spaces.

Local Compactness and One Point Compactification. Stone- Cech Compactification.

Embedding and Metrization. Embedding Lemma and Tychonoff Embedding. The Urysohn Metrization Theorem. The Nagata – Smirnov Metrization Theorem.

Paracompactness : Different types of refinements and their relationships. Paracompactness – definition in terms of locally finite refinement, various characterizations and examples. A. H. Stone's Theorem concerning paracompactness of metric spaces. Partition of unity and Paracompactness. Properties of Paracompactness w.r.to subspaces and products.

Uniform spaces : Definition and examples. Base and subbase of a uniformity . Uniform topology, uniform continuity and product uniformity. Uniformization of topological spaces. Uniform property. Uniformity generated by a family of pseudometrics. Cauchy filter. Completeness of uniform spaces. Completion of uniform spaces. Compactness and uniformity. Uniform cover.

### References

1. General Topology by J. L. Kelley, Van Nostrand
2. General Topology by S. Willard, Addison-Wesley
3. Topology by J. Dugundji, Allyn and Bacon
4. Topology, A first course by J. Munkres, Prentice Hall, India
5. Introduction to topology and modern analysis by G. F. Simmons, McGraw Hill
6. Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.
7. General Topology by Engelking, Polish Scientific Publishers, Warszawa
8. Counter examples in Topology by L. Steen and J. Seebach

### **A13: Advanced Algebra- I (Marks: 50)**

*(To be done in next meeting)*

### **A14: Computational Mathematics- I (Marks: 50)**

*(To be done in next meeting)*

### **MMATOET304: Statistical Methods (Marks: 50)**

**Introduction:** Nature of Statistics, Uses of Statistics, Statistics in relation to other disciplines, Abuses of Statistics.

**Collection & Presentation of data:** Primary data – designing a questionnaire and a schedule. Secondary data – its major sources. Complete enumeration. Controlled experiments, Sample Surveys, Construction of Tables with one or more factors of classification.

#### **Probability Theory:**

Definition of probability: Classical and relative-frequency approach to probability, Kolmogorov's Axiomatic definition, limitations of Classical definition. Probability of union and intersection of events, Conditional probability and Independence of events, Bayes' Theorem and its applications. Examples based on classical approach and repeated trials.

**Bivariate data:** Scatter diagram, correlation coefficient and its properties, Correlation ratio, Correlation Index, Intraclass correlation, Concept of Regression, Principles of least squares, Fitting of polynomial and exponential curves. Rank correlation – Spearman's and Kendall's measures.

**Some Standard Sampling Distributions:**  $\chi^2$  distribution, distributions of the mean and variance of a random sample from a normal population, t and F distributions, distributions of means, variances and correlation coefficient (null case) of a random sample from a bivariate normal population.

#### **Multivariate Analysis**

Multivariate data – multiple regression, multiple correlation and partial correlation – their properties and related results.

**Analysis of Variance:** One way classification. Assumptions regarding model. Two way classification with equal number of observations per cell.

### References

1. Goon AM, Gupta MK, Dasgupta B. (1998): Fundamentals of Statistics (V-1), World Press
2. Yule G.U & Kendall M.G (1950): An Introduction to the Theory of Statistics, C. Griffin
3. Kendall M.G. & Stuart A. (1966): Advanced Theory of Statistics (Vols 1 & 2)
4. Snedecor & Cochran (1967): Statistical Methods (6th ed), Iowa State Univ. Press
5. Croxton F.E., Cowden D.J. & Klein (1969): Applied General Statistics, Prentice Hall.
6. Ross S.M. (1972): Introduction to Probability Models, Academic Press.
7. Bhattacharya GK & Johnson R. A. (1977): Concepts & Methods of Statistics, John Wiley.
8. Mukhopadhyay P. (1999): Applied Statistics.

### MMATCCS305: Computer aided Numerical Practical (Marks: 50)

Numerical and statistical programs using C language

**Sessional** (Working formula, Algorithm and Program with output): 10 marks

**Problem 1:** 25 marks (Algorithm – 10, Program – 10, Result – 5)

#### List of Problems:

1. Inversion of a non-singular square matrix by Gauss-Jordan Method.
2. Solution of a system of linear equations by Gauss-Seidel method.
3. Integration by Romberg's method and Gauss-Legendre quadrature method.
4. Solution of polynomial equations by QD algorithm and Bairstow's method.
5. Solution of an Initial Value problem of first order O.D.E. by Milne's method.
6. Solution of an Initial Value problem of second order O.D.E. and first order simultaneous equations by 4th order Runge-Kutta method.
7. B.V.P. for second order O.D.E. by finite difference method.
8. Solution of parabolic equation in two variables by explicit Schmidt formula and implicit Crank-Nicolson's method.
9. Solution of one dimensional wave equation by finite difference method.
10. Solution of elliptic type PDE.

**Problem 2 (Simple unknown):** 10 marks (Program – 5, Result – 5)

Simple numerical and statistical problems using the C language.

**Viva-voce:** 5 marks

### MMATOPP306: Outreach Programme

In this course, the students will be engaged in different e-learning program and they will share their computer knowledge to the school level/college level students. It will develop their teaching and communicative skill also.

**SEMESTER-IV**

**MMATCCT401: Graph Theory and Mathematical Logic (Marks: 50)**

**Group-A: Graph Theory (Marks: 30)**

**Introduction:** Definition-Graph, Subgraph, Complement, Walks, Paths, cycles, connected components, Cut vertices and cut edges, Distance, radius and center, Diameter, Degree sequence, and its application.

**Trees:** Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles

**Eulerian and Semi Eulerian Graphs. Hamiltonian Graphs.** Maximum edges in a non-hamiltonian graph, Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Traveling salesman problem, Chinese postman problem

**Vertex and edge connectivity.** Chromatic number, Bipartite graph. Broke's Theorem, Mycielski Construction, Chromatic polynomial, edge colouring number,

**Matrix Representation:** Adjacency matrix, Incidence matrix, Cycle rank and co-cycle rank, Fundamental Cycles with respect to Spanning tree and Cayley's theorem on trees.

**Planar graphs:** Statement of Kuratowski Theorem, Isomorphism of graphs, Eulers formula, colour theorem. 4 colour theorem, Dual of a planar Graph.

**Directed Graph:** Binary relations, directed paths, fundamental Circuits in Digraphs, Adjacency matrix of a Digraph.

Searching algorithms-BFS and DFS algorithms. Graph Ramsey theorem.

**References**

1. Bondy and Murty, Graph Theory with Applications (Macmillan, 1976)
2. Deo, N., Graph Theory (Prentice-Hall, 1974)
3. Harary, F., Graph Theory (Addison-Wesley, 1969)
4. Pathasarthi, K. R., Basic Graph Theory (TMH., 1994).
5. J A Bondy and U. S R Murty, Graph Theory, GTM 244 Springer, 2008.
6. Mehdi Behzad and ary Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.
7. Kenneth Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.
8. D.B.West, Introduction to Graph Theory, PHI, 2009.

**Group-B: Mathematical Logic (Marks: 20)**

**Propositional Logic:** Syntax, Unique parsing, Semantics, Equivalences, Consequences, Calculations, Informal proofs.

**Normal Forms and Resolution:** Clauses, CNF and DNF representations, Adequacy of calculations, SAT, Resolution refutation, Adequacy of resolution.

**Proof Systems:** Axiomatic system PC, Adequacy of PC, Analytic tableau PT, Adequacy of PT, Compactness of PL.

**First Order Logic:** Syntax of FL, Scope and binding, Substitutions, Semantics of FL, Quantifier laws, Equivalences, Consequences.

**Normal Forms in FL:** Calculations, Informal proofs, Prenex forms, Skolem forms, Herbrand's theorem, Skolem-Lowenheim theorem, Resolution in FL

**Proof Systems for FL:** Axiomatic system FC, Analytic tableau FT, Adequacy of FC and FT, Compactness in FL.

**Axiomatic Theories:** Undecidability of FL, Godel's incompleteness theorems.

**References**

1. Singh, A., Logics for computer science, PHI, 2004.
2. Singh, A. and Goswami, C., Fundamentals of Logic, ICPR, New Delhi, 1998.
3. Shoenfield, J. R., Mathematical Logic, Addison Wesley, Reading, Massachusetts, 1967.
4. Smullyan, R., First order logic, Springer Verlag, New York, 1968.
5. Elliott Mendelson; Introduction to mathematical logic; Chapman & Hall; London (1997)
6. Angelo Margaris; First order mathematical logic; Dover publications, Inc, New York (1990).
7. S.C.Kleene; Introduction to Metamathematics; Amsterdam; Elsevier (1952).
8. J.H.Gallier; Logic for Computer Science; John.Wiley & Sons (1987).
9. H.B.Enderton; A mathematical introduction to logic; Academic Press; New York (1972).

**MMATCCT402: Operator Theory and Elements of Dynamical System (Marks: 50)**

**Group A: Operator Theory (Marks: 30)**

Conjugate or Dual spaces. Representation of bounded Linear functionals on  $C[a,b]$  and on  $L_p$  spaces. Duals of  $C[a,b]$  &  $L_p$ . Weak & weak\* convergence. Reflexive spaces.

Adjoint of an Operator and its Properties; Normal, Self Adjoint, Unitary, Projection Operators, their Characterizations & Properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum & Product of Projections, Invariant Subspaces.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only).

Existence and Representation of the Inverse of I—T, Representation of the Resolvent Operators, Non-emptiness of the Resolvent Set, Spectral Radius and its Representation, Spectral Mapping Theorem for Polynomials.

**References:**

1. Rudin, Functional Analysis
2. Schaffer, Topological Vector Spaces
3. Bachman & Narici, Functional Analysis
4. Kryszic, Functional Analysis
5. Diestel, Applications of Geometry of Banach Spaces
6. Horvat, Linear Topological spaces

**Group B: Elements of Dynamical System (Marks – 20)**

**Dynamical Systems:** Phase variables and Phase space, continuous and discrete time systems, flows(vector fields), maps (discrete dynamical systems), orbits, asymptotic states, fixed (equilibrium) points periodic points, concepts of stability, dynamical system as a group.

**Linear systems:** Fundamental theorem and its application. Properties of exponential of a matrix, generalized eigenvectors of a matrix, nilpotent matrix, stable, unstable and center subspaces, hyperbolicity, contracting and expanding behaviour.

**Nonlinear Vector Fields:** Stability characteristics of an equilibrium point. Liapunov and asymptotic stability. Source, sink, basin of attraction. Phase plane analysis of simple systems, homoclinic and heteroclinic orbits, hyperbolicity, statement of Hartmann-Grobman theorem.

Liapunov function and Liapunov theorem. Periodic solutions, limit cycles and their stability concepts. Statement of Lienard's theorem and its application to vander Pol equation, Poincare-Bendixsom theorem (statement and applications only), structural stability and bifurcation through examples of saddle-node, pitchfork and Hopf bifurcations.

**References:**

1. P. Glendinning – *Stability, Instability and Chaos* (Cambridge University Press 1994).
2. Strogartz – *Non-linear Dynamics*
3. M. W. Hirsch & S. Smale – *Differential Equations, Dynamical Systems and Linear Algebra* (Academic Press 1974)
4. L. Perko – *Differential Equations and Dynamical Systems* (Springer – 1991)
5. Arnold – *Ordinary Differential Equations*

**MMATECT403 & MMATECT404**

All the students will have to take two Special Papers in Semester IV from the clusters of special papers as given below. The allocation of special papers will be monitored by the Department.

**B1: Advanced Complex Analysis II (Marks: 50)**

**B2: Advanced Functional Analysis II (Marks: 50)**

**B3: Advanced Real Analysis II (Marks: 50)**

Density of arbitrary linear sets. Lebesgue density theorem. Approximate continuity. Properties of approximately continuous functions. Bounded approximately continuous function over  $[a,b]$  and exact derivative. The Perron integral : Definitions and basic properties, Comparison with Lebesgue integral and Newton integral. Trigonometric system and Trigonometric Fourier series. Summability of Fourier series by  $(C, I)$ , means.

Fejer's theorem. Lebesgue's theorem. Completeness of Trigonometric system.

Sets of the 1st and of the 2nd categories. Baire's theorem for  $G_\delta$ , residual and perfect sets, points of condensation of a set.

Baire classification of functions. Functions of Baire class one. Baire's theorem. Semicontinuous functions.

**References:**

1. Hewitt and Stormberg, Real and Abstract Analysis
2. Royden, H. L., Real Analysis
3. Saks, Theory of Integrals
4. Rudin, W., Real and Abstract Analysis
5. Munroe, M. E., Measure and Integration

**B4: Advanced Algebraic Topology II (Marks: 50)**

Introduction of singular homology and cohomology group by Eilenberg and Steenrod axioms. Existence and Uniqueness of singular homology and cohomology theory.

Calculation of homology and cohomology groups for circle. Projective spaces, torus relation between  $H_1(X)$  and  $\pi_1(X)$ .

Singular cohomology ring, calculation of cohomology ring for projective spaces.

Fibre bundles: Definitions and examples of bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles, homotopy properties of vector bundles. Homology exact sequence of a fibre bundle.

**References :**

1. Massey, W. S., Algebraic Topology
2. Massey, W. S., Singular Homology Theory
3. Spanier, E. H., Algebraic Topology
4. Gray, B., Homotopy Theory An Introduction to Algebraic Topology

**B5: Advanced Differential Geometry of Manifolds-II (Marks: 50)**

Bundle: Tangent bundle, Vector bundle, Subbundle, fibre bundle, Bundle homomorphisms. Co-vector, Co-tangent Bundle.

Riemannian manifolds as metric space: Riemannian distance function, tangent-cotangent isomorphism, Pseudo-Riemannian metric

Differential Form: Differential 1-form, Characterisation of 1-form, Pullback of 1-form, Differential r-form, Pull-Back Differential r- Form.

Tensor fields: Tensors on a vector space, Tensor fields, mapping and covariant tensors, multiplication of tensor fields, Symmetrizing and alternating transformations, Exterior Algebra, Exterior Differentiation

Differentiation on Riemannian manifolds: Differentiation of vector fields, Connections, Riemannian manifold, Riemannian connection, Fundamental theorem of Riemannian geometry.

Curvature: Curvature tensor, Riemann Curvature tensor, Sectional curvature, Schur's theorem, projective Curvature tensor, Ricci Curvature, scalar Curvature

Geodesic in a Riemannian Manifold

**References:**

1. Kobayashi, S. & Nomizu, K., Foundation of differential Geometry, vol-1.
2. Boothby, W. M., An Introduction to Differentiable Manifolds and Riemannian Geometry.
3. Auslander, L. & Mackenzie, R. E., Introduction to Differentiable Manifolds.
4. Chern, S. S., Chen, W. H. & Lam, K. S., Lectures on Differential Geometry.
5. K. YANO, K.M. KON—Differentiable Manifold.
6. D.E. BLAIR—Contact Manifolds in Riemannian Geometry, Lecture Notes in Maths.
7. Loring W. Tu- Differential geometry
8. J.M. Lee- Differential Geometry

**B6: Advanced Computational Fluid Dynamics- II (Marks: 50)**

Conservation principles of fluid dynamics, Basic Equations of viscous and inviscid flow, Basic equation in conservation form, Boundary conditions.

Condition for Euler and Navier-Stokes' equations, Grid generation using elliptic partial differential equations, Boundary-Layer equations, Incompressible viscous flow field computation, Spatial and temporal discretization on collocated and on staggered grids, Stream function vorticity and MAC method, Implementation of boundary conditions, Turbulence modeling, Viscous compressible flow computation based on RANS using simple turbulence modeling.



**References:**

1. Niyogi, P., Chakraborty, S. K. and Laha, M. K., Introduction to Computational Fluid Dynamics, Pearson education, Delhi, 2005.
2. Fletcher, C. A. J., Computational Techniques for Fluid Dynamics, Vol-1 and Vol-II, Springer.
3. Peyret, R., and Taylor, T. D., Computational Methods for Fluid Flow, Springer 1983.
4. Thompson, J.F., Warsi, Z.U.A. and C.W. Martin, C.W., Numerical Grid Generation, Foundation and Applications, North Holland 1985
5. Landau, L. D., Lifshitz, E. M., Fluid Mechanics, Trans, Pergamon Press 1989
6. Schlichting, H., Gersten, K., Boundary – Layer Theory, 8<sup>th</sup> Ed., Springer 2000

**B7: Advanced Fluid Mechanics II (Marks: 50)**

Basic thermodynamics of compressible fluids, Equation of state, Conservation of energy, Euler's equation of motion, Propagation of small disturbances in a gas, Dynamics similarity of two flows, Steady isentropic motion, Bernoulli's equation, Subsonic and supersonic flows, Plane rotational and irrotational motion with supersonic velocity, Steady flow through a De Level nozzle, Normal and oblique shock wave shock polar diagram.

Characteristics and their use for solution of plane irrotational problem, Prandtl-Mayer flow past a convex cone.

Steady linearized subsonic and supersonic flows, Prandtl-Glauert transformation, Flow along a wavy boundary, Flow past a slight corner, Janzen-Rayleigh method of approximation, Ackeret's formula.

Lagendre and Molenbroek transformations Chaplygin's equation for stream function, Solution of Chaplygin's equation, Subsonic and supersonic flow past a thin bodies, Limiting line, Motion due to a two dimensional source and vortex, Karman-Tsien approximation, Transonic flow, Euler's-Tricomi equation and its fundamental solution, Hypersonic flow.

**Reference:**

1. Thompson, P. A., Compressible fluid dynamics.
2. Shaprou, A. H., Compressible fluid flow.
3. Lipman, B., Aspects of subsonic and transonic flows.
4. Niyogi, P., Inviscid gas dynamics, Mcmillan, 1975 (India)
5. Oswatitsch, K., Gas dynamics.

**B8: Advanced Viscous Flows, Boundary Layer Theory and**

**Magneto-hydrodynamics II (Marks - 50)**

Physical description of electrically conducting fluids, Maxwell's electromagnetic field equations, Mass, Momentum & Energy conservation laws in MHD, Basic MHD equations: Equation of continuity, Equations of motion, Energy equation, Lorentz force, Ohm's law (including & excluding Hall current) for a static and for a moving conductor.

MHD approximations and simplifications of Generalised Ampere's law, Lorentz force and total current due to these approximations.

Equation of induction, Dimensionless forms of equations of motion and induction, Lundquist criterion, Convection dominated case for equation of induction.

Alfven's theorem and its interpretation, Diffusion dominated case of induction equation, Lorentz force and its physical interpretation, Alfven waves, Poynting theorem.

Parallel steady flow of viscous, incompressible fluid, unidirectional flow under a transverse magnetic field, Hartmann flow, Couette flow.

Unsteady unidirectional motion (MHD Rayleigh problem) of viscous, incompressible fluid.

Magnetohydrostatics, Pinch effect, Linear pinch, Stability of pinch configurations.

Force free-field and its general solution, Non-existence of force-free field of finite extent, Toroidal and Poloidal fields.

Dynamo theory, Symmetric fields, Cowling's theorem, Concept of isorotation, Ferraro's law of isorotation.

#### References:

1. Ferraro, V.C.A. & Plumpton, C., An introduction to Magneto-Fluid Mechanics, Clarendon Press, 1966.
2. Cowling, T.G., Magnetohydrodynamics, Interscience Publishers Ltd., 1956.
3. Shercliff, J.A., A text book of Magnetohydrodynamics, Pergaman Press, 1965.

#### **B9: Advanced Operations Research II (Marks: 50)**

**Dynamic Programming:** Characteristics of Dynamic Programming problems, Bellman's principle of optimality (Mathematical formulation)

Model –1: Single additive constraint, multiplicative separable return,

Model – 2: Single additive constraint, additively separable return,

Model – 3: Single a multiplicative constraint, additively separable return,

Multistage decision process – Forward and Backward recursive approach, Dynamic Programming approach for solving linear and non-linear programming problems, Application – Single-item N-period deterministic inventory model.

**Geometric Programming:** Elementary properties of Geometric Programming and its applications. Geometric programming for unconstrained objective function and constrained objective function.

**Queuing Theory:** Introduction, characteristic of Queuing systems, operating characteristics of Queuing

system. Probability distribution in Queuing systems. Classification of Queuing models. Poisson and non Poisson queuing models  $(M/M/1:\infty/FCFS/\infty)$ ,  $(M/M/C:\infty/FCFS/\infty)$ ,  $(M/M/1:N/FCFS/\infty)$ ,  $(M/M/C:N/FCFS/\infty)$ .

**Reliability theory:** Definition, failure rate, Hazard rate, series arrangement, parallel arrangement, reliability evaluation.

**References:**

1. Sharma, J. K., Operations Research – Theory and Applications
2. Taha, Operations Research
3. Schaum’s Outline Series – Operations Research
4. Swarup, Gupta & Manmohan, Operations Research
5. Kapoor, V.K., Operations Research
6. Rao, S.S., Optimization theory and Applications, Wiley Eastern Ltd., New Delhi.
7. Bector, Chandra and Dutta, Principles of optimization Theory, Narosa Publishing House.

**B10: Advanced Quantum Mechanics II (Marks: 50)**

**Collision Theory:** Basic concepts, Cross sections, Laboratory and center-of-mass coordinates, Rutherford scattering, Quantum mechanical formulation - time independent and time-dependent, Scattering of a particle by a short-range potential, Scattering by Coulomb potential, Scattering by screened Coulomb field, Scattering by complex potential.

**Integral Equation Formulation:** Lippmann-Schwinger integral equation and its formal solutions, Integral representation of the scattering amplitude, Convergence of the Born Series, Validity of Born approximation, Transition probabilities and cross sections.

**Semi-Classical Approximations:** WKB approximation, Eikonal approximation.

**Variational Principles in the Theory of Collisions:** General formulation of the variational principle, Hulthen, Kohn-Hulthen and Schwinger variational methods, Determination of Phase shifts, Scattering length and scattering amplitude for central force problems, Bound (minimum) principles.

**Analytic Properties of Scattering Amplitude:** Jost function, Scattering matrix, Bound states and resonances, Levinson theorem, Dispersion relations, Effective range theory.

**References:**

1. Messiah, A., Quantum Mechanics, Vol. I & II (North – Holland Pub. Co., 1962).
2. Bransden, B. H. & Joachain, C. J., Introduction to Quantum Mechanics (Oxford University Press, 1989).
3. Burke, P. G., Potential Scattering in Atomic Physics (Plenum Press, New York, 1977)
4. Joachain, C. J., Quantum Collision Theory (North-Holland Pub. Co., 1975)
5. Bransden, B. H., Atomic Collision Theory (W. A. Benjamin Inc., N. Y., 1970)
6. Geltman, S., Topics in Atomic Collision Theory (Academic Press. 1969)
7. Wu, T. Y. and Olmura, T., Quantum Theory of Scattering (Prentice Hall, New Jersey, 1962)
8. Mott, N. F. & Massey, H. S. W., Theory of Atomic collisions (3rd ed.), (Clarendon Press, Oxford, 1965)

9. Goldberger, M. L. & Watson, K. M., Collision Theory (Wiley, N. Y., 1964)
10. Newton, R. G., Scattering Theory of Waves and Particles (McGraw – Hill, 1966)

**B11: Advanced Theory of Elasticity-II (Marks: 50)**

Propagation of waves in an isotropic elastic solid medium, Motion of a surface of discontinuity: kinematical condition and dynamical conditions, Kirchoff's solution of inhomogeneous wave equation, Waves of dilation and distortion, Plane waves, Elastic surface waves such as Rayleigh, Stonely and Love waves, Dispersion and group velocity of elastic body waves.

Plane stress and plane strain problems, Solution of plane stress and plane strain problems in polar coordinates, General solution for an infinite plate with a circular hole, An infinite plate under the action of concentrated forces and moments.

Beam stretched by its own weight, Solution of differential equations of equilibrium in terms of stresses, Stress function, Reduction of Lamé and Beltrami equations to biharmonic equations, Relvin and Boussineq-Papkovich solution, Pressure on the surface of a semi-infinite body.

Theory of thin plates, Basic equations for bending of plates and boundary conditions, Navier's and Levy solutions for rectangular plates, Circular plate, Cylindrical bending of uniformly loaded plates.

**References:**

1. Love, A.E.H., A Treatise on the Mathematical Theory of Elasticity, Dover.
2. Sokolnikoff, I.S., Mathematical Theory of Elasticity, McGraw Hill, 1956.
3. Fung, Y.C., Foundations of Solid Mechanics.
4. Eringen, A.C., Elasto-dynamics.
5. Sada, A.S., Elasticity theory and Applications.
6. Chung, T.J., Continuum Mechanics, Prentice Hall, 1988.

**B12: Advanced Topology- II (Marks: 50)**

**Function Spaces:** Pointwise convergence; Uniform convergence. The compact open Topology, Uniform convergence on compacta. The Stone-Weierstrass Theorem.

**Rings of continuous functions:** The ring  $C(X)$  & its subring  $C^*(X)$ , their Lattice structure. Ring homomorphism and lattice homomorphism.

Zero-sets, cozero-sets, completely separated sets and its characterization,  $C$ -embedding &  $C^*$  embedding and their relation, Urysohn's extension theorem. Characterizations of Normal spaces and Pseudocompact spaces in terms of  $C$ -embedding &  $C^*$  embedding.

Adequacy of Tychonoff  $X$  for consideration of  $C(X)$ ,  $C^*(X)$  – M.H. Stones theorem.  $Z$ -filters,  $Z$ -ultrafilters on  $X$ , their duality with ideals,  $Z$ -ideals and maximal ideals of  $C(X)$ . Structure spaces of  $C(X)$ ,  $C^*(X)$ , hull-kernel topology. Banach-Stone theorem. Wallman compactification, the partially ordered set  $K(X)$  of all compactifications of  $X$ , its lattice structure. Gelfand-Kolmogoroff theorem. Variant constructions of  $\beta X$  achieved as, (i) The structure space of  $C(X)$ , (ii) The structure space of  $C^*(X)$ , (iii) The space of all nonzero real valued homomorphisms on  $C^*(X)$ . The spaces  $\beta N$ ,  $\beta Q$  and  $\beta R$ .

**References**

- [1] Gillman and Jerison, Rings of continuous functions; Springer-verlag, N.Y. Heidelberg, Berlin, 1976.
- [2] Charles E. Aull, Rings of continuous functions; Marcel Dekker. Inc. 1985.

- [3] R. C. Walker, The Stone-Ćech compactification; Springer, N.Y. 1974.
- [4] R. E. Chandler, Hausdorff compactifications; Marcel Dekker, Inc. N.Y. 1976.
- [5] J. Dugundji, Topology; Boston, Allyn and Bacon, 1966.
- [6] Porter and Woods, Extensions and Absolutes of Hausdorff spaces; Springer, 1988.
- [7] Franklin Mendivil, Function Algebras & The Lattice of Compactifications; Proceedings of the American Mathematical Society, Vol-127, No. 6, Pages: 1863-1871, 1999.
- [8] J. L. Kelley, General Topology, Van Nostrand
- [9] S. Willard, General Topology, Addison-Wesley

**B13: Advanced Algebra- II (Marks: 50)**

*(To be done in the next meeting)*

**B14: Computational Mathematics- II (Marks: 50)**

*(To be done in the next meeting)*

**MMATACT405: Computer Application (Marks: 50)**

Word processing, Excel spreadsheet, Power Point Presentation, Latex, Simple programming in C, Graph plotting using Excel, Origin, MATLAB.

**MMATMEP406: Project (Marks: 50)**

The project work and/or group project work will be performed on some advanced topics related to special papers or any advance topic or review work of research papers. The marks distribution of project work is as follows: 20 marks are allotted for written submission, 20 marks are for seminar presentation and 10 marks are allotted for viva-voce examination.